Equation of State
at finite baryon density and external magnetic field
from Lattice QCD

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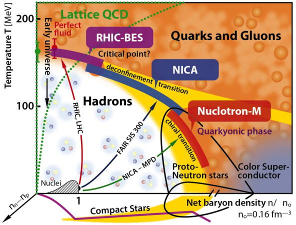
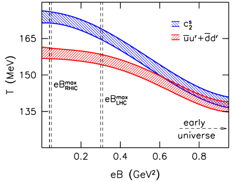


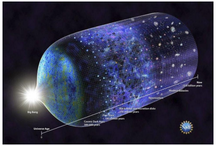
http://nica.jinr.ru/physics.php



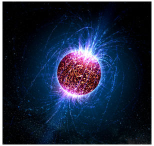
G. Endrodi, J.Phys.Conf.Ser. **503** (2014) 012009 [arXiv:1311.0648 [hep-lat]]

External magnetic field influences phase diagram!

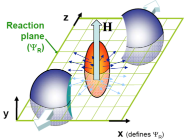
High density & Strong magnetic field



Early Universe



Neutron stars



Heavy Ion Collisions

QCD on the lattice

Lattice allows calculate path integral and expectation values of observables:

$$\boxed{\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U]\mathcal{D}[\psi,\bar\psi]\, \mathrm{e}^{-S(\psi,\bar\psi,U)}\, \mathcal{O}(\psi,\bar\psi,U),} \qquad \mathcal{Z} = \int \mathcal{D}[U]\mathcal{D}[\psi,\bar\psi]\, \mathrm{e}^{-S(\psi,\bar\psi,U)}$$

$$\mathcal{D}[U] = \prod_{n,\mu} \mathrm{d}U_\mu(n), \quad \mathcal{D}[\psi,\bar\psi] = \prod_{\substack{n,f\\ \alpha,a}} \mathrm{d}\psi^{(f)}_{\substack{\alpha\\ a}}(n)\, \mathrm{d}\bar\psi^{(f)}_{\substack{\alpha\\ a}}(n).$$

Lattice: Euclidean; hypercubic; with spacing a.

Action S with its parameters; initial configuration $\{U,\ \psi,\ \bar\psi\}_0$ → MC simulations on the lattice → Boltzmann ensemble of configurations $\{U,\ \psi,\ \bar\psi\}_k$

$$\boxed{\langle \mathcal{O} \rangle \approx \frac{1}{K} \sum_{k=1}^{K} \mathcal{O}\left[\{U,\ \psi,\ \bar\psi\}_k\right],} \qquad \{U,\ \psi,\ \bar\psi\}_k \sim \mathrm{e}^{-S(\psi,\bar\psi,U)}$$

Measured quantities: $\mathcal{O} = \mathrm{Tr} \prod_{n,\mu\in\mathcal{C}} U_\mu(n), \quad \bar\psi(n)U_\mu(n)\psi(n+\hat\mu), \quad \bar\psi(n)\Gamma\psi(n),\ \ldots$

Continuous limit $a \to 0$ should be taken!

Thermodynamics on the lattice

$$p = -\frac{\Omega}{V} = \frac{T}{V}\ln\mathcal{Z} \qquad \longleftarrow \text{ cannot be measured directly}$$

$$/\star \quad \mathcal{Z} = \int \mathcal{D}[U]\mathcal{D}[\psi,\bar\psi]\,\mathrm{e}^{-S(\psi,\bar\psi,U)} \quad \star/$$

Derivatives of p can be measured!

$$n_q = \frac{N_q}{V} = \frac{\partial p}{\partial \mu_q} \qquad \text{– quark number density}$$

$$n_q = \frac{T}{V}\cdot\frac{1}{\mathcal{Z}}\cdot\frac{\partial\mathcal{Z}}{\partial\mu_q} = -\frac{T}{V}\cdot\underbrace{\frac{1}{\mathcal{Z}}\int\mathcal{D}[U]\mathcal{D}[\psi,\bar\psi]\left(\frac{\partial S}{\partial\mu_q}\right)\mathrm{e}^{-S(\psi,\bar\psi,U)}}_{\left\langle\frac{\partial S}{\partial\mu_q}\right\rangle} = -\frac{T}{V}\left\langle\frac{\partial S}{\partial\mu_q}\right\rangle$$

$$\boxed{\begin{aligned} \mu_B &= \mu_u + 2\mu_d \\ \mu_Q &= \mu_u - \mu_d \\ \mu_S &= \mu_d - \mu_s \\ \\ n_B &= (n_u + n_d + n_s)/3 \\ n_Q &= (2n_u - n_d - n_s)/3 \\ n_S &= -n_s \end{aligned}}$$

J. N. Günther et al., Nucl. Phys. A **967**, 720 (2017) [arXiv:1607.02493 [hep-lat]]

$$\frac{p}{T^4} = c_0(T) + c_2(T)\left(\frac{\mu_B}{T}\right)^2 + c_4(T)\left(\frac{\mu_B}{T}\right)^4 + c_6(T)\left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

$$\frac{n}{\mu_B T^2} = \frac{T}{\mu_B}\cdot\frac{d(p/T^4)}{d(\mu_B/T)} = 2c_2 + 4c_4\left(\frac{\mu_B}{T}\right)^2 + 6c_6\left(\frac{\mu_B}{T}\right)^4 \qquad \leftarrow \text{ coefficients can be found from fit}$$

(μ_u, μ_d, μ_s are such that $\langle n_S\rangle = 0$, $\langle n_Q\rangle = 0.4\langle n_B\rangle$)

Calculation of c_0:

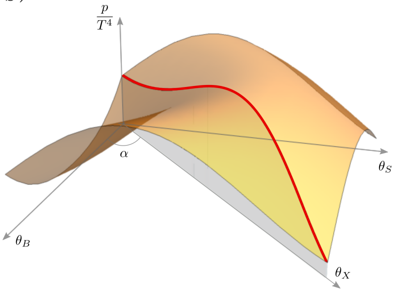
S. Borsanyi et al., Phys. Lett. B **730**, 99 (2014) [arXiv:1309.5258 [hep-lat]].
G. S. Bali et al., JHEP **08**, 177 (2014) [arXiv:1406.0269 [hep-lat]].

Expansion with our choice of chemical potentials

Our choice of chemical potentials:

$$\mu_u = \mu_d = \mu_q; \quad \mu_s = 0 \qquad \Rightarrow \qquad \underbrace{\mu_B = 3\mu_q; \quad \mu_Q = 0; \quad \mu_S = \mu_q}_{\text{1D section of 3D surface } p(\mu_B, \mu_Q, \mu_S)}$$

We want to reconstruct EoS on this section



Dimensionless chemical potential: $\theta = \mu/T$

Axes rotation ($\tan\alpha = \theta_S/\theta_B = 1/3$):

$$\theta_X = (3\,\theta_B + \theta_S)/\sqrt{10} = \theta_q\sqrt{10}$$

Correspondent density:

$$n_X = (3n_B + n_S)/\sqrt{10} = (n_u + n_d)/\sqrt{10}$$

Expansion of the pressure on the section: $$\frac{p}{T^4} = \frac{p_0}{T^4} + c_2^X\theta_X^2 + c_4^X\theta_X^4 + c_6^X\theta_X^6 + \mathcal{O}(\theta_X^8)$$

$c_2^X,\ c_4^X,\ c_6^X$

Expansion of the density: $$\frac{n_X}{T^3} = \frac{\partial(p/T^4)}{\partial\theta_X} = 2c_2^X\theta_X + 4c_4^X\theta_X^3 + 6c_6^X\theta_X^5.$$

Lattice setup

- Tree level improved Symanzik gauge action.
- Staggered $2+1$ fermionic action.
- Stout smearing improvement.
- Imaginary chemical potential: $\mu = i\mu_I$.
- External magnetic field:

$$\vec{B} = B\vec{e}_z; \qquad B = \text{const} \qquad\qquad A_y^{\text{ext}} = Bx/2, \quad A_x^{\text{ext}} = -By/2, \quad A_\mu^{\text{ext}} = 0, \;\; \mu = z, t$$

- Splitting of the rooted determinant:

$$\mathcal{Z} = \int \mathcal{D}U \, e^{-S_G} [\det D(B, m_u, q_u)]^{\frac{1}{4}} [\det D(B, m_d, q_d)]^{\frac{1}{4}} [\det D(B, m_s, q_s)]^{\frac{1}{4}}$$

$$D(n|f) = \frac{1}{2a} \sum_\mu \eta_\mu(n) \left[u_\mu(B,q,n)\, \Xi_\mu \, U_\mu(n) \delta_{f,n+\hat{\mu}} - u^\star_\mu(B,q,f)\, \Xi^\star_\mu \, U^\dagger_\mu(f) \delta_{f,n-\hat{\mu}} \right] + m\, \delta_{f,n}$$

$$u_x(B,q,n_x,n_y,n_z,n_t) = e^{-ia^2 qB\, n_y/2}, \quad n_x \neq N_x - 1, \qquad u_y(B,q,n_x,n_y,n_z,n_t) = e^{ia^2 qB\, n_x/2}, \quad n_y \neq N_y - 1,$$

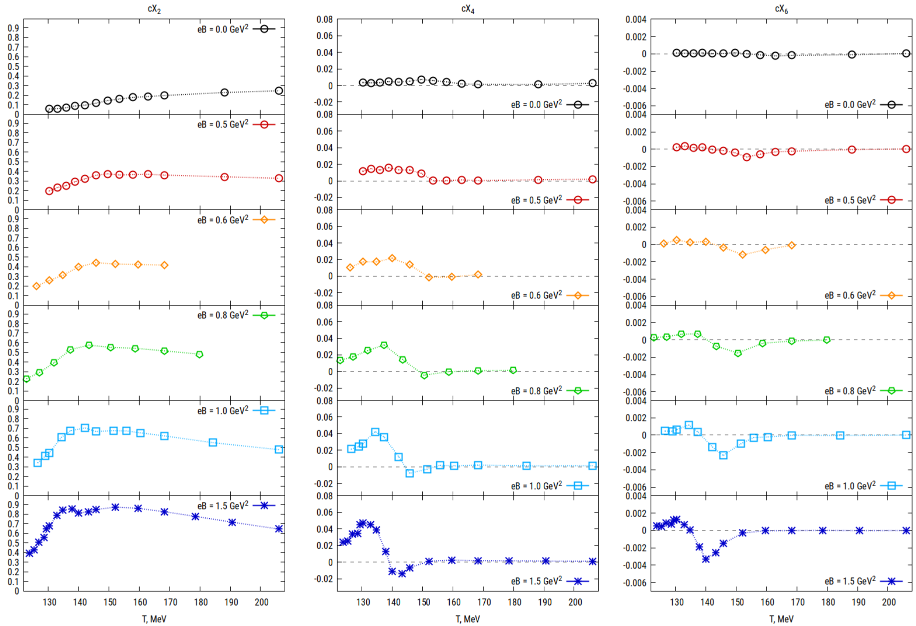
$$u_x(B,q,N_x-1,n_y,n_z,n_t) = e^{-ia^2 qB(N_x+1)n_y/2}, \qquad u_y(B,q,n_x,N_y-1,n_z,n_t) = e^{ia^2 qB(N_y+1)n_x/2}.$$

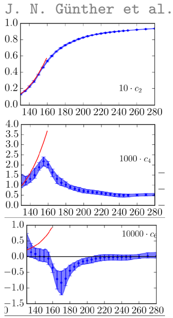
$$\Xi_\nu = e^{ia\mu_I \times \delta_{\nu 4}}$$

Periodic boundary conditions $\Rightarrow$ $eB = \dfrac{6\pi k}{N_x N_y a^2}$, $\quad k \in \mathbb{Z}$ $\qquad$ /* $\; T = \dfrac{1}{N_t \cdot a} \;$ */

Simulation parameters: 6×24^3 lattice;
$eB = 0.0,\ 0.5,\ 0.6,\ 0.8,\ 1.0,\ 1.5$ GeV^2;
$T = 123 - 206$ MeV;
physical quark masses.

Results: coefficients

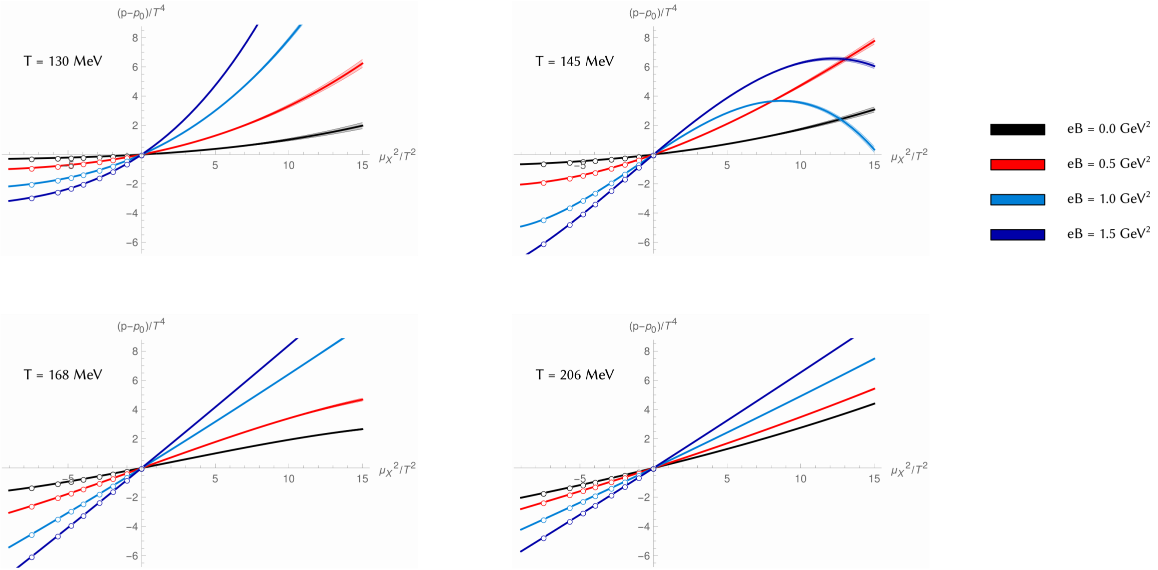




$eB = 0$ GeV^2

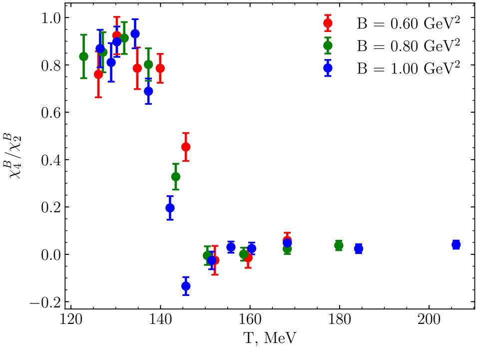
cont. extrapolated

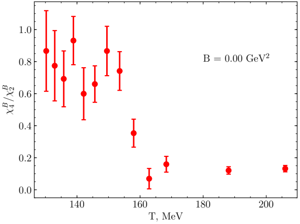
Results: pressure

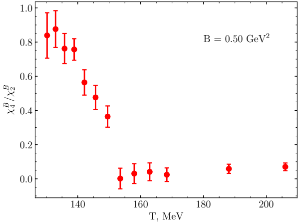


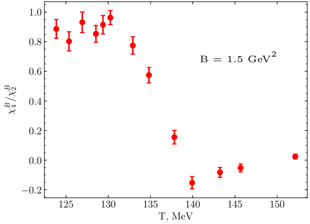
Results: cumulants

$$\chi_B^{(k)} = \frac{\partial^k (p/T^4)}{\partial \theta_B^k}$$









Conclusions

- Monte Carlo simulations are carried out on 6×24^3 lattice in presence of non-zero chemical potential and with external magnetic field.
- Contribution of chemical potential to the pressure is obtained for various temperatures and external field strengths.
- Temperature dependences of cumulant relations are obtained for various values of applied field.
- Strong dependence of the EoS on magnetic field is observed.

Plans for future:

- Perform simulations on larger lattices and take continuum limit.

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Thank you for your attention!