Phase diagram and dualities in two color QCD

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IZMIRAN, IHEP

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Russian Science Foundation

БАЗИС
Фонд развития теоретической физики и математики

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in the broad sense our group stems from
Department of Theoretical Physics, Moscow State University
Prof. V. Ch. Zhukovsky

details can be found in

Eur.Phys.J.C 80 (2020) 10, 995 arXiv:2005.05488 [hep-ph]
JHEP 06 (2020) 148 arXiv:2003.10562 [hep-ph]
Phys.Rev. D100 (2019) no.3, 034009 arXiv: 1904.07151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

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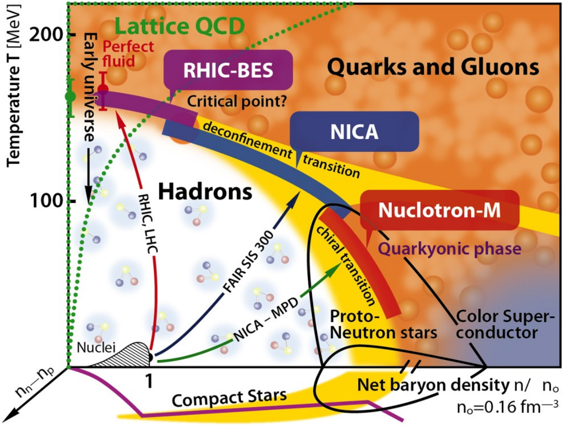
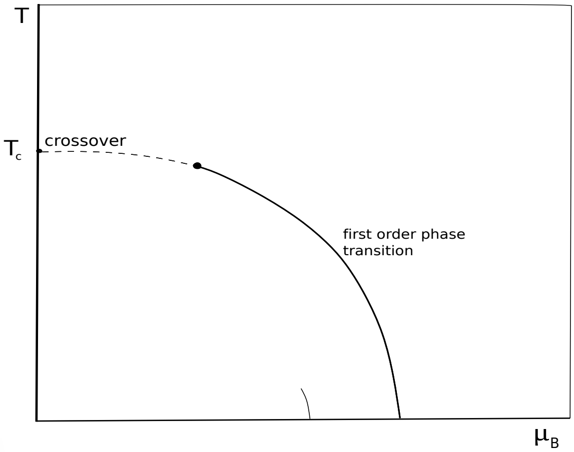
QCD Phase Diagram



Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

QCD Phase Diagram



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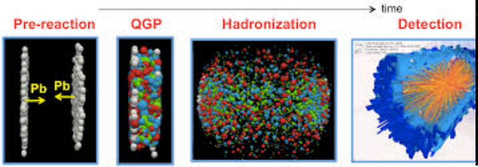
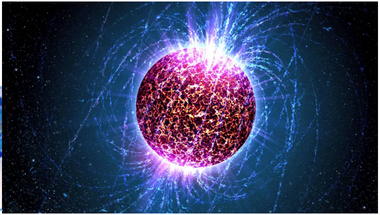
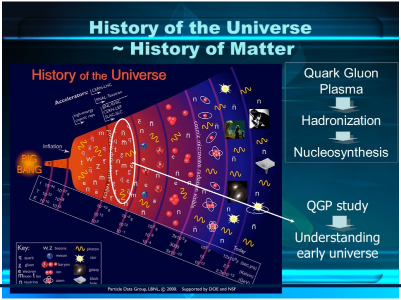
QCD at extreme conditions

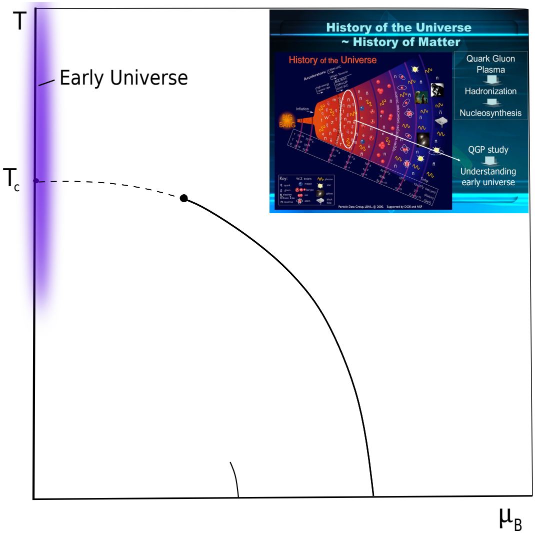






QCD at T and μ
(QCD at extreme conditions)

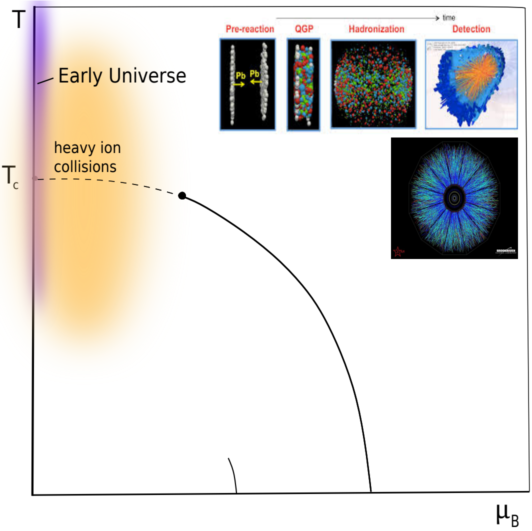
- Early Universe



QCD Dhase Diagram and Methods

QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions



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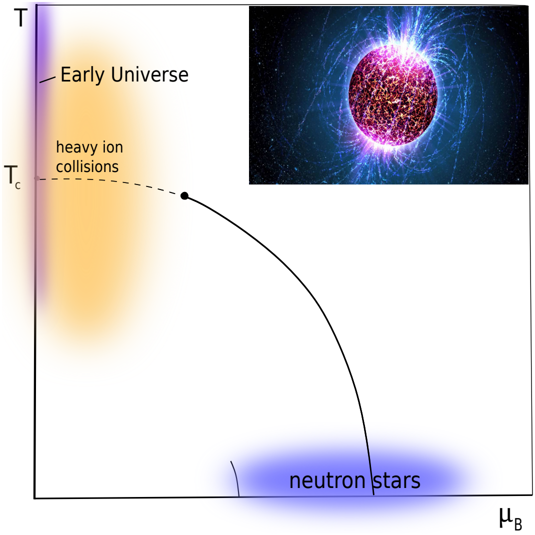

QCD at T and μ
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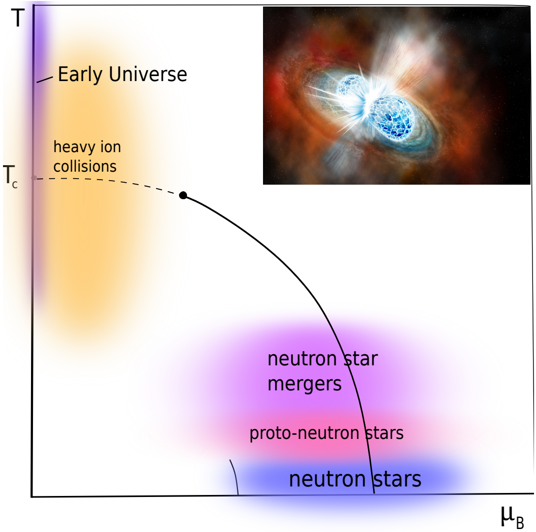
- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



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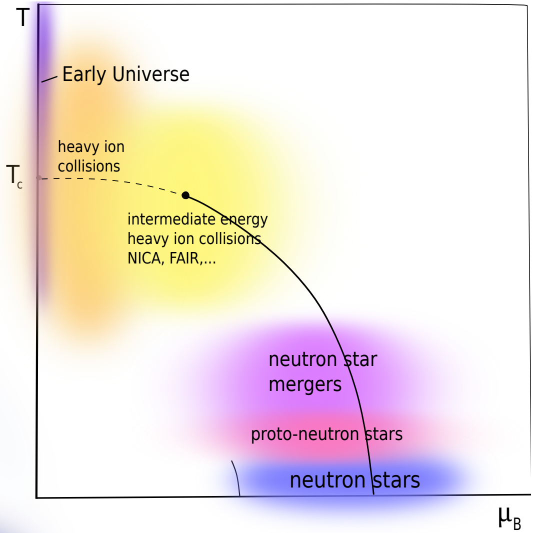
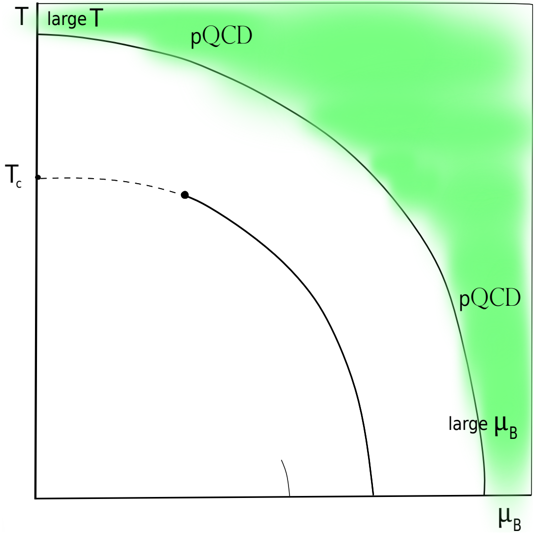


Methods of dealing with QCD

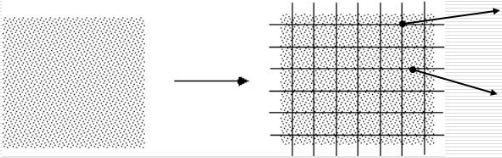
- Perturbative QCD



QCD on a space-time lattice

K. G. Wilson 1974

Space-time continuum   Space-time lattice



- Feynman path integral
  - Action $S_{QCD} = \frac{1}{g_s^2} \sum_P tr(UUUU) + \sum_f \bar{q}_f (\gamma \cdot U + m_f) q_f$
  - Physical quantities as integral averages

$$\langle O(U, \bar{q}, q) \rangle = \frac{1}{Z} \int \prod_{n\mu} dU_{n\mu} \prod_n d\bar{q}_n dq_n \; O(U, (U, \bar{q}, q)) e^{-S_{QCD}}$$

Monte Carlo Evaluation of the path integral

$$Z = \int D[\text{gluons}]\, D[\text{quarks}]\, e^{-S^E_{QCD}}$$

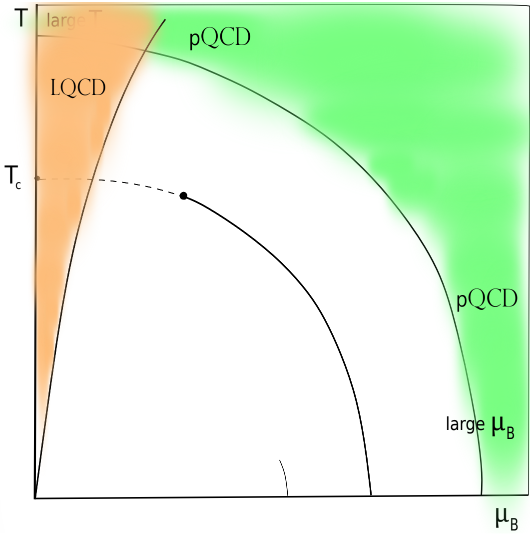
$$Z = \int D[\text{gluons}]\, Det\, \mathbb{D}(\mu)\, e^{-S^E_{\text{gluons}}}$$

It is well known that **at non-zero baryon chemical potential μ_B lattice simulation** is quite challenging due to the **sign problem**
complex determinant

$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

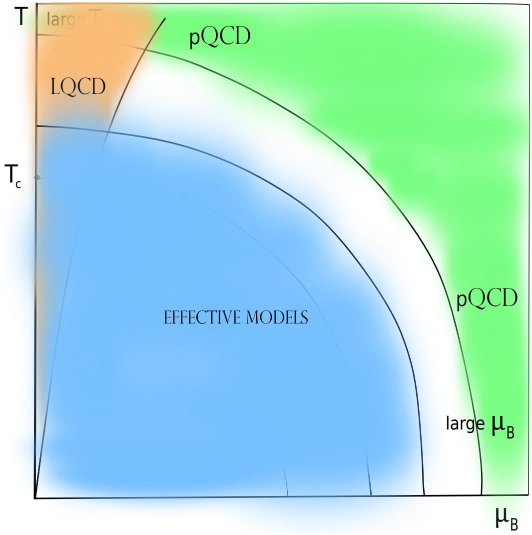
Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD



Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- .....



Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\Big]$$

$$q \to e^{i\gamma_5\alpha}q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi^2\Big].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

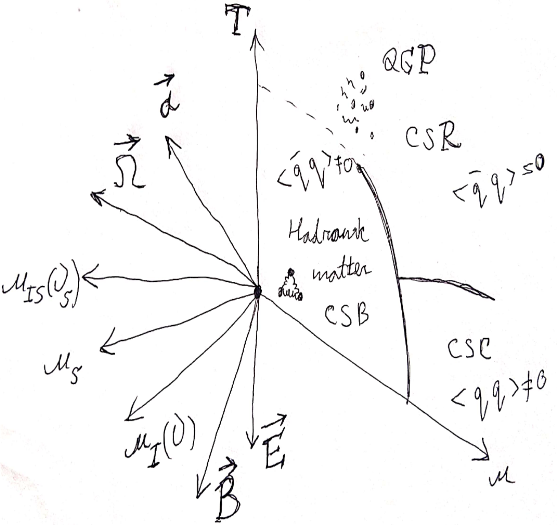
$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\Big]q$$

More external conditions to QCD

More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



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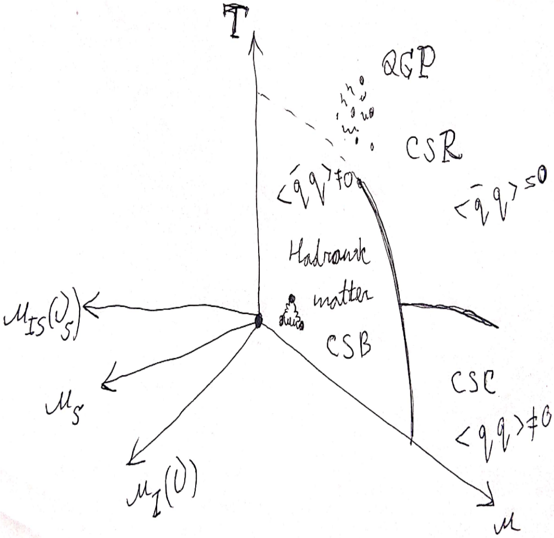


Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad\qquad n_B = \frac{1}{3}(n_u + n_d)$$

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

chiral (axial) chemical potential

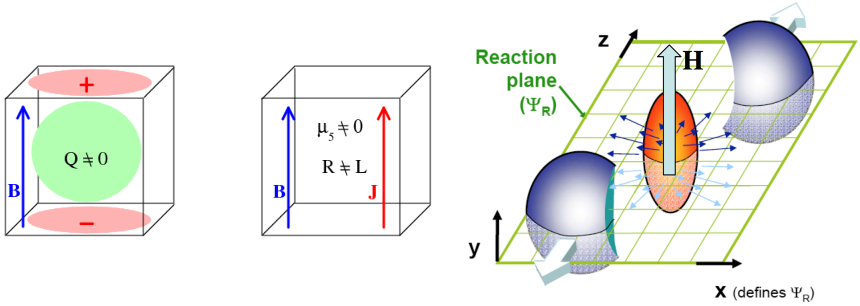
Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

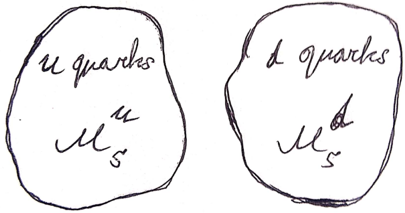
Chiral magnetic effect



$$\vec{J} \sim \mu_5 \vec{B},$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033

Chiral isospin imbalance



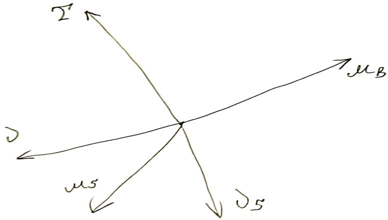
$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \quad \longleftrightarrow \quad \nu_5$$

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$



Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

$$M = \langle \sigma(x) \rangle \sim \langle \bar{q} q \rangle, \qquad \text{CSB phase:} \quad M \neq 0,$$

$$\pi_1 = \langle \pi_1(x) \rangle = \langle \bar{q} \gamma^5 \tau_1 q \rangle, \qquad \text{PC phase:} \quad \pi_1 \neq 0,$$

The TDP

$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...)$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...) \qquad\qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad\qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

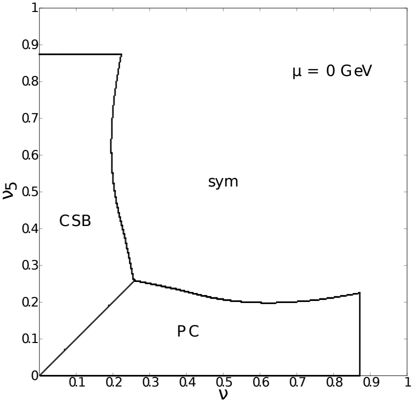


Figure: NJL model results

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\mathrm{PC} \longleftrightarrow \mathrm{CSB} \quad \nu \longleftrightarrow \nu_5$$

Dualities on the lattice

$(\mu_B, \mu_I, \mu_{I5}, \mu_5)$

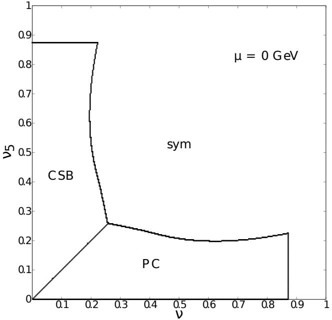
$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

- **QCD at** μ_5 —- (μ_5, T)

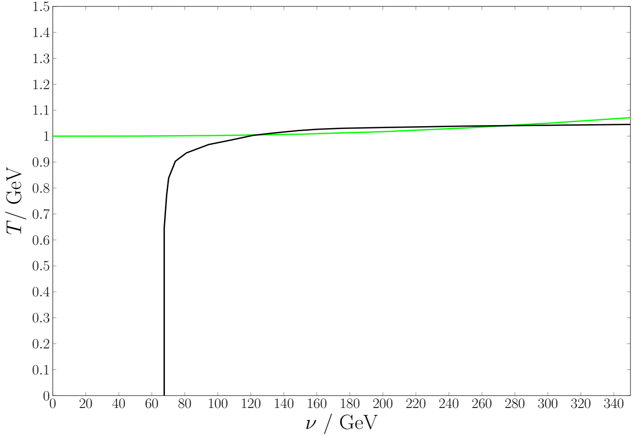
  V. Braguta, A. Kotov et al, ITEP lattice group

- **QCD at** μ_I —- (μ_I, T)

  G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()



Dualities on the lattice



T_c^M as a function of μ_5 (green line) and $T_c^\pi(\nu)$ (black)

Uses of Dualities

How (if at all) it can be used

Let us discuss only Inhomogeneous phases (case)

discussed in Particles 2020, 3(1), 62-79

schematic (ν_5, μ)-phase diagram

$$(\mu, \nu) \longrightarrow (\mu, \nu_5)$$

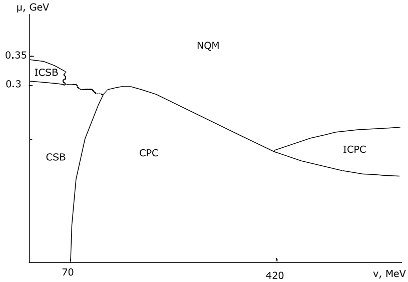


Figure: (ν, μ)-phase diagram.

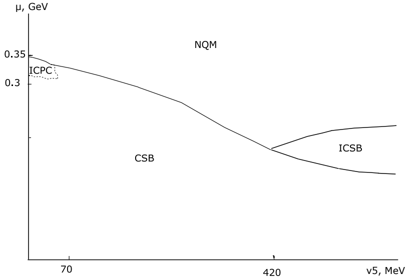


Figure: (ν_5, μ)-phase diagram

M. Buballa, S. Carignano, J. Wambach, D. Nowakovski, Lianyi He et al.

Two colour QCD case

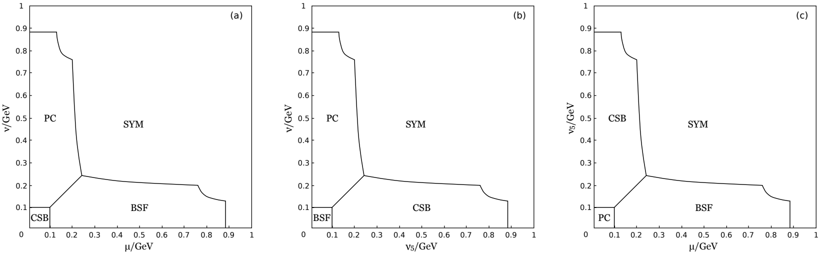
QC_2D

Condensates and phases

$$M = \langle \sigma(x) \rangle \sim \langle \bar{q}q \rangle, \qquad \text{CSB phase:} \quad M \neq 0,$$

$$\pi_1 = \langle \pi_1(x) \rangle = \langle \bar{q}\gamma^5 \tau_1 q \rangle, \qquad \text{PC phase:} \quad \pi_1 \neq 0,$$

$$\Delta = \langle \Delta(x) \rangle = \langle qq \rangle = \langle q^T C \gamma^5 \sigma_2 \tau_2 q \rangle, \qquad \text{BSF phase:} \quad \Delta \neq 0.$$



$$(a) \qquad \mathcal{D}_1: \quad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|, \qquad \text{PC} \longleftrightarrow \text{BSF}$$

J. Andersen, T. Brauner, D. T. Son, M. Stephanov, J. Kogut, ...

$$(b) \qquad \mathcal{D}_3: \quad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \qquad \text{PC} \longleftrightarrow \text{CSB}$$

$$(c) \qquad \mathcal{D}_2: \quad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|, \qquad \text{CSB} \longleftrightarrow \text{BSF}$$

Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of NJL model

- In the mean field approximation

Instead of chiral symmetry

$$SU_L(2) \times SU_R(2)$$

one has Pauli-Gursey flavor symmetry

$$SU(4)$$

Two colour NJL model

$$L = \bar{q}\Big[i\hat{\partial} - m_0\Big]q + H\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \big(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\big)\big(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\big)\Big]$$

Dualities are connected with Pauli-Gursey group

$$\mathcal{D}_3: \quad \psi_R \to i\tau_1\psi_R$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$$\bar{\psi}\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_1\psi$$

$$M \longleftrightarrow \Delta, \qquad \nu \longleftrightarrow \nu_5, \quad \mu_I \longleftrightarrow \mu_{I5}$$

$$i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi \leftrightarrow i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi, \quad \bar{\psi}^C\sigma_2\tau_2\psi \leftrightarrow \bar{\psi}^C\sigma_2\tau_2\psi$$

$$\bar{\psi}\tau_2\psi \leftrightarrow \bar{\psi}\tau_3\psi, \quad \bar{\psi}\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^5\psi, \quad i\bar{\psi}\gamma^5\tau_2\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_3\psi$$

There is also $\mathcal{D}_1$ and $\mathcal{D}_2$

Dualities are connected with Pauli-Gursey group

Dualities were found in

- In the framework of NJL model beyond mean field

- In QC$_2$D non-pertubartively (at the level of Lagrangian)

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Duality was found in

- In the framework of NJL model beyond mean field or at all orders of N_c approximation

- In QCD non-pertubartively (at the level of Lagrangian)

Conclusions and Thanks for the Attention

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied in two color color case
- It was shown that there exist dualities in QCD and $\mathrm{QC_2D}$

  Richer structure of Dualities in the two colour case
- There have been shown ideas how dualities can be used

  Duality is not just entertaining mathematical property but an instrument with very high predictivity power
- Dualities have been shown non-perturbetively in the two colour case
- Duality has been shown non-perturbarively in QCD