Collective oscillations of plasma and order parameter in graphene counterflow superconductors.

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Introduction

In the BCS theory the superconductive state is described by the complex order parameter $\Delta = |\Delta|e^{i\varphi}$. Cooper pairing of electrons results in appearing of the gap $2|\Delta|$ in the energy spectrum. In the normal state $\Delta = 0$. In the superconductive state $\Delta \neq 0$.

There are specific collective excitations in systems with Cooper pairing:

- Anderson–Bogoliubov (AB) mode (oscillations of phase of the order parameter)
- Schmid mode (oscillations of modulus of the order parameter)
- Carlson–Goldman (CG) mode (coupled oscillations of phase of the order parameter and electromagnetic field)

These modes exist under certain conditions.

The genuine AB mode can exist in neutral superfluids.

The CG mode can exist only in a narrow temperature range near T_c, and it has strong damping in "clean" superconductors.

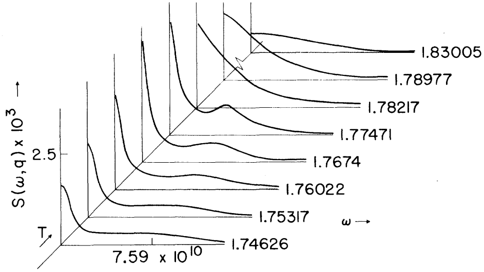
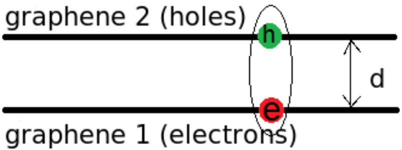


Figure: The CG mode in superconductive aluminium films. [R. V. Carlson, A. M. Goldman, PRL **34**, 11 (1975)]

We consider the double layer graphene system, which is an example of so-called counterflow superconductors.



The system consists of two parallel graphene layers, which are separated by a thin dielectric. The layers have different types of conductivity (an electron and a hole ones).

Electrons and holes from the opposite layers may form pairs due to Coulomb attraction.

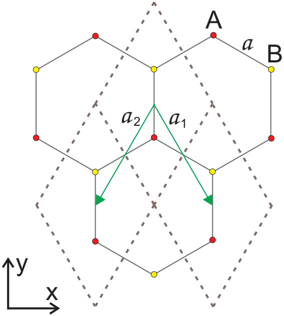
2D Bose gas of electron-hole pairs may transit into superluid state under $T < T_c$.

Supefluid motion of pairs corresponds to electron supercurrents in layers, which are equal by modulus, but opposite in direction.

It is shown that the analogs of AB, Schmid, and CG modes appear in counterflow superconductors.

The Hamiltonian of double layer graphene system

$$\hat{H} = -t \sum_{n,<i,j>,\sigma} \left[\hat{c}^{+}_{n,i,A,\sigma} \hat{c}_{n,j,B,\sigma} + h.c. \right] + U \sum_{i,l,\sigma} \hat{c}^{+}_{1,i,l,\sigma} \hat{c}^{+}_{2,i,l,\sigma} \hat{c}_{2,i,l,\sigma} \hat{c}_{1,i,l,\sigma} - \\ - \sum_{n,i,l,\sigma} \left[e\varphi_{n,i}(t) + \mu_n \right] \hat{c}^{+}_{n,i,l,\sigma} \hat{c}_{n,i,l,\sigma}.$$



$n = 1, 2$ – layer, i – unit cell, $l = A, B$ – sub-lattice, $\sigma =\uparrow, \downarrow$ – spin,

μ_n – chemical potential ($\mu_1 = \mu$, $\mu_2 = -\mu$),

φ – scalar potential of electromagnetic field,

t – nearest-neighbor "hopping" parameter,

$U = const > 0$ – Coulomb interaction energy of two electrons.

Here $< i, j >$ means the summation over nearest neighbor unit cells:
$\mathbf{R}_j \to \mathbf{R}_i + \delta_{m=0,1,2}$, $\delta_0 = 0$, $\delta_1 = \mathbf{a}_1$, $\delta_2 = \mathbf{a}_2$.

We introduce the order parameter of electron-hole pairing as follows:

$$\Delta_{i,\sigma} = U\langle \hat{c}^+_{2,i,A,\sigma}\hat{c}_{1,i,A,\sigma}\rangle = -U\langle \hat{c}^+_{2,i,B,\sigma}\hat{c}_{1,i,B,\sigma}\rangle.$$

The order parameter can be splitted into equilibrium part and fluctuations:

$$\Delta_{i,\sigma} = \Delta + \Delta^{(fl)}_{i,\sigma}(t).$$

In the momentum representation we have:

$$\hat{H} = \hat{H}_0 + \hat{H}_{int}, \qquad \hat{H}_0 = \sum_{\mathbf{k},\sigma} \hat{\Psi}^+_{\mathbf{k},\sigma}\hat{h}_{\mathbf{k}}\hat{\Psi}_{\mathbf{k},\sigma}, \qquad f_{\mathbf{k}} = |f_{\mathbf{k}}|e^{i\chi_{\mathbf{k}}} = -t\sum_m e^{i\mathbf{k}\delta_m}.$$

$$\hat{\Psi}_{\mathbf{k},\sigma} = \begin{pmatrix} \hat{c}_{1,A,\mathbf{k},\sigma} \\ \hat{c}_{1,B,\mathbf{k},\sigma} \\ \hat{c}_{2,A,\mathbf{k},\sigma} \\ \hat{c}_{2,B,\mathbf{k},\sigma} \end{pmatrix}, \qquad \hat{h}_{\mathbf{k}} = \begin{pmatrix} -\mu & f_{\mathbf{k}} & -\Delta & 0 \\ f^*_{\mathbf{k}} & -\mu & 0 & \Delta \\ -\Delta & 0 & \mu & f_{\mathbf{k}} \\ 0 & \Delta & f^*_{\mathbf{k}} & \mu \end{pmatrix}.$$

Hamiltonian of interaction:

$$\hat{H}_{int}(t) = -\frac{1}{2\pi S}\sum_{\mathbf{k},\mathbf{q},\sigma}\int d\omega e^{-i\omega t}\hat{\Psi}^+_{\mathbf{k}+\mathbf{q},\sigma} \times$$

$$\times\left[\frac{e}{2}\varphi_+(\mathbf{q},\omega)\hat{T}^{(0)} + \Delta_{1,\sigma}(\mathbf{q},\omega)\hat{T}^{(1)} + \Delta_{2,\sigma}(\mathbf{q},\omega)\hat{T}^{(2)} + \frac{e}{2}\varphi_-(\mathbf{q},\omega)\hat{T}^{(3)}\right]\hat{\Psi}_{\mathbf{k},\sigma}.$$

$\Delta_{1(2),\sigma}$ – real and imaginary parts of the order parameter fluctuations,
$\varphi_\pm = \varphi_1 \pm \varphi_2$.

Response functions

The Hamiltonian $\hat{H}_0$ can be diagonalized:

$$\hat{H}_0 = \sum_{\nu} E_\nu \hat{a}_\nu^+ \hat{a}_\nu, \qquad E_\nu = mE_{\mathbf{k},\lambda}, \qquad E_{\mathbf{k},\lambda} = \sqrt{\xi_{\mathbf{k},\lambda}^2 + \Delta^2}, \qquad \xi_{\mathbf{k},\lambda} = \lambda|f_{\mathbf{k}}| - \mu.$$

$\hat{a}_\nu^+$, $\hat{a}_\nu$ – operators of creation and annihilation of quasiparticles in the state $\nu = \{m = \pm 1; \mathbf{k}; \lambda = \pm 1; \sigma\}$.

To calculate the response of the system to the scalar potential and to the order parameter oscillations, one may define the response functions:

$$\eta_\sigma^{(s)}(\mathbf{q},\omega) = \int dt e^{i\omega t} \sum_{\mathbf{k}} \langle \hat{\Psi}_{\mathbf{k}-\mathbf{q},\sigma}^+ \hat{T}^{(s)} \hat{\Psi}_{\mathbf{k},\sigma} \rangle = \int dt e^{i\omega t} \sum_{\mathbf{k}} Sp \left(\hat{\rho}(t) \hat{\Psi}_{\mathbf{k}-\mathbf{q},\sigma}^+ \hat{T}^{(s)} \hat{\Psi}_{\mathbf{k},\sigma} \right).$$

The density matrix $\hat{\rho}(t)$ satisfies the Liouville–von Neumann equation:

$$\frac{\partial \hat{\rho}(t)}{\partial t} = \frac{1}{i\hbar}[\hat{H}(t), \hat{\rho}(t)] - \gamma(\hat{\rho}(t) - \hat{\rho}_0), \qquad \hat{\rho}(t) = \hat{\rho}_0 + \hat{\rho}_1(t) + ...$$

The solution can be found as a series in $\hat{H}_{int}$. Self-consistence equation for Δ:

$$\Delta = \frac{2g}{S} \sum_{\mathbf{k},\lambda} \frac{\Delta}{2E_{\mathbf{k},\lambda}} \tanh \frac{E_{\mathbf{k},\lambda}}{2T},$$

$g = U\Omega_0/4$ – coupling constant, Ω_0 – unit cell area.
The response of the system is calculated in the linear approximation.

System of equations for eigenmode spectrum

$$\begin{pmatrix} 2\Pi_{00} - \frac{2}{V_+(q)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \Pi_{11}^{(R)} & \Pi_{12} & 0 & 0 & \Pi_{13} \\ 0 & \Pi_{21} & \Pi_{22}^{(R)} & 0 & 0 & \Pi_{23} \\ 0 & 0 & 0 & \Pi_{11}^{(R)} & \Pi_{12} & \Pi_{13} \\ 0 & 0 & 0 & \Pi_{21} & \Pi_{22}^{(R)} & \Pi_{23} \\ 0 & \Pi_{31} & \Pi_{32} & \Pi_{31} & \Pi_{32} & 2\Pi_{33} - \frac{2}{V_-(q)} \end{pmatrix} \begin{pmatrix} e\varphi_+(\mathbf{q},\omega)/2 \\ \Delta_{1,\uparrow}(\mathbf{q},\omega) \\ \Delta_{2,\uparrow}(\mathbf{q},\omega) \\ \Delta_{1,\downarrow}(\mathbf{q},\omega) \\ \Delta_{2,\downarrow}(\mathbf{q},\omega) \\ e\varphi_-(\mathbf{q},\omega)/2 \end{pmatrix} = 0,$$

$$\Pi_{11}^{(R)} = \Pi_{11} + \frac{1}{g}, \qquad \Pi_{22}^{(R)} = \Pi_{22} + \frac{1}{g},$$

$$V_\pm(q) = V_{11} \pm V_{12} = \frac{4\pi e^2}{q} \frac{1 \pm e^{-qd}}{(\varepsilon+1) \mp (\varepsilon-1)e^{-qd}}.$$

Π_{ij} – polarization functions of the first order.

Equating the determinant to zero, we obtain the dispersion relations for collective modes.

Polarization functions

$$\Pi_{s_1,s_2} = \frac{1}{S}\sum_{\nu_1,\nu_2}\delta_{\mathbf{k}_1-\mathbf{q},\mathbf{k}_2}\Phi^{s_1 s_2}_{\nu_1\nu_2}\frac{1+\lambda_1\lambda_2\cos\left(\chi_{\mathbf{k}_1}-\chi_{\mathbf{k}_2}\right)}{2}\frac{n_F(E_{\nu_1})-n_F(E_{\nu_2})}{E_{\nu_1}-E_{\nu_2}-\hbar(\omega+i\gamma)},$$

where

$$\Phi^{00}_{\nu_1\nu_2} = \frac{1}{2}\left(1+\frac{\xi_1\xi_2+\Delta^2}{E_1E_2}\right), \qquad \Phi^{01}_{\nu_1\nu_2} = -\frac{\Delta}{2}\left(\frac{1}{E_1}+\frac{1}{E_2}\right),$$

$$\Phi^{02}_{\nu_1\nu_2} = i\frac{\Delta}{2}\frac{\xi_2-\xi_1}{E_2E_1}, \quad \Phi^{03}_{\nu_1\nu_2} = \frac{1}{2}\left(\frac{\xi_2}{E_2}+\frac{\xi_1}{E_1}\right), \quad \Phi^{11}_{\nu_1\nu_2} = \frac{1}{2}\left(1-\frac{\xi_1\xi_2-\Delta^2}{E_1E_2}\right),$$

$$\Phi^{12}_{\nu_1\nu_2} = \frac{i}{2}\left(\frac{\xi_1}{E_1}-\frac{\xi_2}{E_2}\right), \qquad \Phi^{13}_{\nu_1\nu_2} = -\frac{\Delta}{2}\frac{\xi_1+\xi_2}{E_1E_2},$$

$$\Phi^{22}_{\nu_1\nu_2} = \frac{1}{2}\left(1-\frac{\xi_1\xi_2+\Delta^2}{E_1E_2}\right), \qquad \Phi^{23}_{\nu_1\nu_2} = i\frac{\Delta}{2}\left(\frac{1}{E_2}-\frac{1}{E_1}\right),$$

$$\Phi^{33}_{\nu_1\nu_2} = \frac{1}{2}\left(1+\frac{\xi_1\xi_2-\Delta^2}{E_1E_2}\right), \qquad \Phi^{s_2 s_1}_{\nu_1\nu_2} = \left(\Phi^{s_1 s_2}_{\nu_1\nu_2}\right)^*$$

$\nu = \{m, \mathbf{k}, \lambda, \sigma\}$ – quasiparticle quantum numbers.

The AB and Schmid modes

Dispersion equation and energy loss function:

$$\Pi_{11}^{(R)}(\mathbf{q},\omega)\Pi_{22}^{(R)}(\mathbf{q},\omega) + [\Pi_{12}(\mathbf{q},\omega)]^2 = 0.$$

$$L_{11}(\mathbf{q},\omega) = \frac{1}{g} Im \left[\frac{1}{\Pi_{11}^{(R)}(\mathbf{q},\omega) + \frac{[\Pi_{12}(\mathbf{q},\omega)]^2}{\Pi_{22}^{(R)}(\mathbf{q},\omega)}} \right],$$

$$L_{22}(\mathbf{q},\omega) = \frac{1}{g} Im \left[\frac{1}{\Pi_{22}^{(R)}(\mathbf{q},\omega) + \frac{[\Pi_{12}(\mathbf{q},\omega)]^2}{\Pi_{11}^{(R)}(\mathbf{q},\omega)}} \right],$$

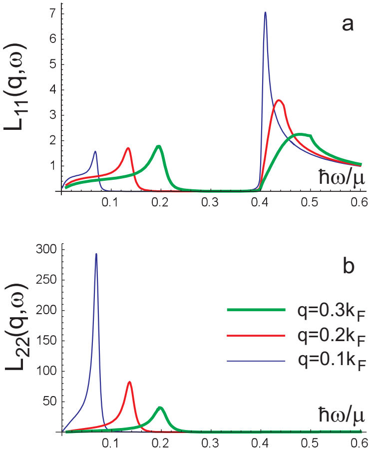
This equation describes the out-of-phase oscillations of order parameters of two spin subsystems $\Delta_\uparrow - \Delta_\downarrow$.

Such oscillations are uncoupled from scalar potential φ.

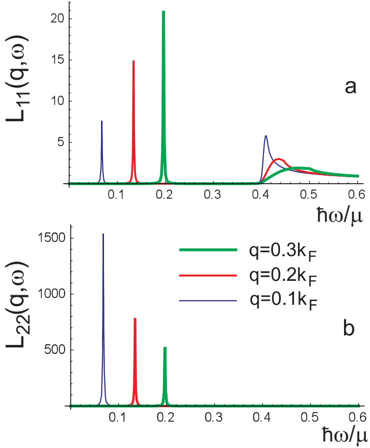
L_{11} and L_{22} correspond to oscillations of modulus and phase of the order parameter, correspondingly.

Frequency dependence of functions L_{11} and L_{22} for $\Delta = 0.2\mu$:

$T = 0.1\mu$



$T = 0$

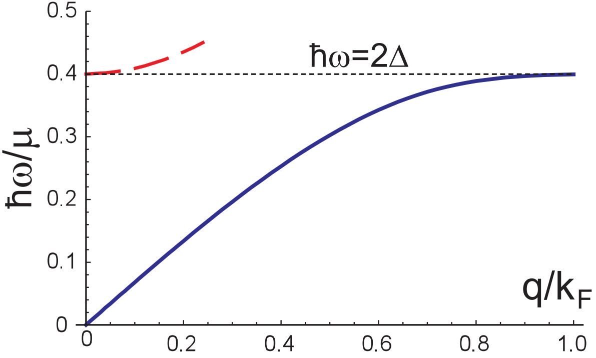


Peaks correspond to eigenmodes, and half-width determines the damping.

AB mode: $\hbar\omega < 2\Delta$;

Schmid mode: $\hbar\omega > 2\Delta$.

Spectrum of Anderson–Bogoliubov and Schmid modes:



$T = 0$, $\Delta = 0.2\mu$

AB mode – lower branch, Schmid mode – upper branch.

At small q the AB mode has an acoustic dispersion: $\omega \approx qv_F/\sqrt{2}$.

Schmid mode exists only at small q.

The Carlson–Goldman mode

Dispersion equation and energy loss function:

$$\varepsilon_-(\mathbf{q},\omega) = 1 - V_-(q)\Pi_-(\mathbf{q},\omega) = 0,$$

where

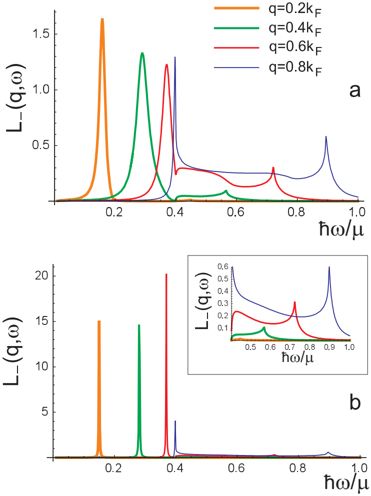
$$\Pi_-(\mathbf{q},\omega) = \Pi_{33}(\mathbf{q},\omega) +$$
$$+\frac{\Pi_{11}^{(R)}(\mathbf{q},\omega)[\Pi_{23}(\mathbf{q},\omega)]^2 - \Pi_{22}^{(R)}(\mathbf{q},\omega)[\Pi_{13}(\mathbf{q},\omega)]^2 + 2\Pi_{12}(\mathbf{q},\omega)\Pi_{13}(\mathbf{q},\omega)\Pi_{23}(\mathbf{q},\omega)}{\Pi_{11}^{(R)}(\mathbf{q},\omega)\Pi_{22}^{(R)}(\mathbf{q},\omega) + [\Pi_{12}(\mathbf{q},\omega)]^2}$$

– polarization function, "dressed" with the order parameter fluctuations.

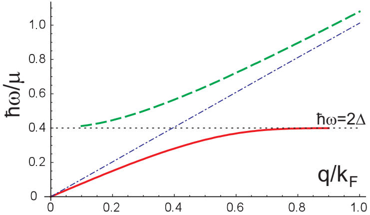
$$L_-(\mathbf{q},\omega) = -Im\left[\frac{1}{\varepsilon_-(\mathbf{q},\omega)}\right]$$

This equation describes in-phase oscillations of order parameters of two spin subsystems coupled to the out-of-phase oscillations of scalar potentials φ_-.

a) $T = 0.1\mu$ b) $T = 0$ $\varepsilon = 4$, $dk_F = 0.1$, $\Delta = 0.2\mu$



Spectrum of CG mode ($T = 0$):



$\hbar\omega < 2\Delta$: CG mode.

$\hbar\omega > 2\Delta$: Strongly damped mode (at $\Delta = 0$ it transforms to acoustic plasmon mode).

The CG mode exists at all $T < T_c$ (unlike of ordinary superconductors).

Optical plasmon mode

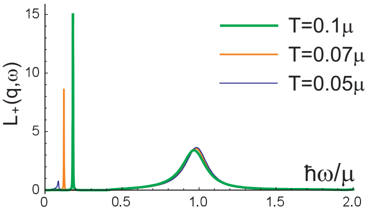
Dispersion equation and energy loss function:

$$\varepsilon_+(\mathbf{q},\omega) = 1 - V_+(q)\Pi_{00}(\mathbf{q},\omega) = 0.$$

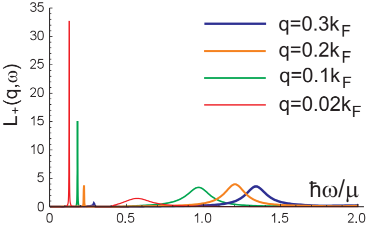
$$L_+(\mathbf{q},\omega) = -Im\left[\frac{1}{\varepsilon_+(\mathbf{q},\omega)}\right].$$

This equation describes the optical plasmon mode, which corresponds to the in-phase oscillations of scalar potentials in layers φ_+.

Such oscillations are uncoupled from the order parameter fluctuations.

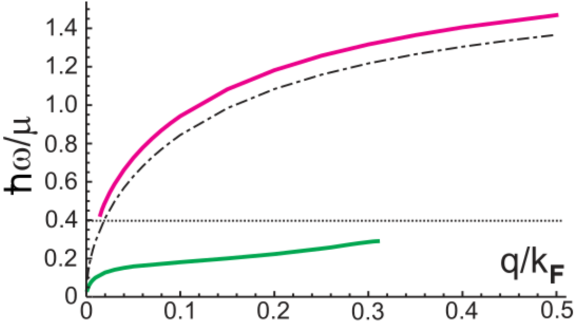


$q = 0.1k_F,\ \Delta = 0.2\mu$



$T = 0.1\mu,\ \Delta = 0.2\mu$

Spectrum of the optical plasmon mode:



$\Delta = 0.2\mu,\ T = 0.1\mu$

This mode splits into two branches: lower branch ($\hbar\omega < 2\Delta$, weakly damped) and upper branch ($\hbar\omega > 2\Delta$, strongly damped).

Lower branch is sensitive to the temperature, and it disappears at $T = 0$.

Conclusions

- It is shown that the Anderson–Bogoliubov, Schmid, and Carlson–Goldman modes appear in the counterflow superconductors.
- The AB mode corresponds to out-of-phase oscillations of order parameters of two spin subsystems. Such oscillations are uncoupled from scalar potential.
- The CG mode corresponds to in-phase oscillations of order parameters of two spin subsystems hybridized with out-of-phase oscillations of scalar potentials in layers. Unlike of ordinary superconductors, the CG mode in counterflow superconductors exists at all $T < T_c$ (and at $T = 0$).
- The optical plasmon mode corresponds to in-phase oscillations of scalar potentials in layers. This mode is uncoupled from order parameter fluctuations, and splits into two branches: lower ($\hbar\omega < 2\Delta$, weakly damped) and upper ($\hbar\omega > 2\Delta$, strongly damped). Lower branch is sensitive to the temperature and disappears at $T = 0$.