On relaxation processes in plasma

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The plasma is considered in a generalized Lorentz model which contrary to standard one assumes that ions form an equilibrium system. Following to Lorentz it is neglected by electron-electron and ion-ion interactions. Relaxation of the electron energy and momentum densities is investigated in spatially uniform states of completely ionized plasma in the presence of small constant and spatially homogeneous external electric field. The kinetic equation is given by the formula:

$$\left[\partial_t \mathrm{f}_p(t) = -F_n \frac{\partial \mathrm{f}_p(t)}{\partial p_n} + I_p(\mathrm{f}_{p'}(t))\right], (F_n = -eE_n, \int d^3 p \mathrm{f}_p(t) = n), (1)$$

where E_n is external electric field, $-e$ is charge of an electron, n is electron density. Perturbation theory is created in terms of spectral theory of operator of collision integral $\mathbf{K}$, which could be defined as $\mathbf{K}a_p = -w_p^{-1} I_p(w_{p'} a_{p'})$ (w_p is Maxwell distribution; $I_p(w_{p'}) = 0$). Linear operator $\mathbf{K}$ is a symmetric and positively defined one. Complete orthonormal system of its own functions g_{ip} could be used to find solutions of kinetic equation (1) as series by mods:

$$\mathrm{f}_p = w_p(1 + g_p), g_p = \sum_i c_i g_{ip}. (2)$$

We use irreducible polynomials as our own functions. The scalar A_p and vector $B_p p_l$ eigenfunctions and corresponding eigenvalues λ_T, λ_u play a decisive role among its own functions

$$\mathbf{K}A_p = \lambda_T A_p, \mathbf{K}B_p p_l = \lambda_u B_p p_l \ (\langle A_p \varepsilon_p \rangle \equiv 3n/2, \langle B_p \varepsilon_p \rangle \equiv 3n/2). (3)$$

It is convenient to investigate the relaxation processes in the system in terms of average electron energy ε and momentum π_l densities. It is established that their evolution is exact described at all times by scalar and vector modes

$$[\varepsilon = \varepsilon_0 + c_T 3n/2], [\pi_l = mnc_{u_l}], (\varepsilon_0 \equiv 3nT/2), (4)$$

where c_T, c_{u_l} – coefficients in series (2) with its eigenfunctions A_p, $B_p p_l$ (m – electron mass, T_0 – ion system temperature). It is proved that quantities ε i π_l at all times and for an arbitrary external electric field E_n satisfy the equation:

$$[\partial_t \pi_l = nF_l - \lambda_u \pi_l], \left[\partial_t \varepsilon = \frac{1}{m}\pi_l F_l - \lambda_T(\varepsilon - \varepsilon_0)\right]. (5)$$

The results (4), (5) were found by using irreducible tensors as eigenfunctions of the operator $\mathbf{K}$. Formulas (5) show that eigenvalues λ_T, λ_u describe relaxation process in the absence of external electric field

$$\varepsilon \to \varepsilon_0, \text{ when } t \gg \tau_T \text{ and } \pi_l \to 0, \text{ when } t \gg \tau_u, (\tau_T \equiv 1/\lambda_T, \tau_u \equiv 1/\lambda_u). (6)$$

In terms of temperature T and velocity u_l of electron system

$$\varepsilon \equiv 3nT/2 + mnu^2/2, \pi_l \equiv mnu_l (7)$$

equations (5) take the form

$$\partial_t T = -\lambda_T(T - T_0) + (2\lambda_u - \lambda_T)mu^2/3, \partial_t u_n = -\lambda_u u_n + \frac{1}{m}F_n (8)$$

These equations are exact and valid at all times and arbitrary electric field. The first one does not contains the electric field. At equilibrium equation (8) gives

$$u_l(t) \underset{t \gg \tau_T, \tau_u}{=} u_l^{eq}, u_l^{eq} = -\nu E_l, \nu \equiv \frac{e}{m\lambda_u}; j_l^{eq} = \sigma E_l, \sigma \equiv \frac{e^2 n}{\lambda_u};$$

$$T(t) \underset{t \gg \tau_T, \tau_u}{=} T^{eq}, T^{eq} = T_0 + \Delta T, \Delta T \equiv \frac{e^2(2\lambda_u - \lambda_T)}{3m\lambda_T \lambda_u^2}E^2 (9)$$

The expression for the mobility of electrons ν and the plasma conductivity σ in (9) are exact. The last formula accurately describes the effect of temperature differences between the electron and ion components of the plasma in equilibrium in the presence of an electric field. This effect was previously discussed in [2] as an approximate result and without accuracy control.

[1] Sokolovsky A.I., Sokolovsky S.A., Hrinishyn O.A. On relaxation processes in a completely ionized plasma. East European Journal of Physics. Vol. 3 (2020). P. 19-30; doi.org / 10.26565/2312-4334-2020-3-03.

[2] Smirnov B.M. Kinetika elektronov v gazakh i kondensirovannykh sistemakh. UFN Vol. 172 (12) (2002). – P. 1411-1445 (in Russian).

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