Critical point and Bose-Einstein condensation in pion matter

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Plan

- Ideal Pion Gas
- Interacting Pion Matter
- Lattice QCD data at zero temperature
- 1st Order Phase Transition, Critical Point, and Bose-Einstein Condensation
- Electric Charge Fluctuations

IDEAL PION GAS

IdPG of three pion species is described in the grand canonical ensemble by the pressure function:

$$P_{\mathrm{id}}(T,\mu) = \sum_{i=+,0,-} p_{\mathrm{id}}(T,\mu_i) = \frac{1}{6\pi^2}\sum_{i=+,0,-}\int_0^{+\infty} f_k(T,\mu_i)\frac{k^4 dk}{\sqrt{k^2+m^2}} \quad (1)$$

Density function:

$$n^{\mathrm{id}}(T,\mu) = \sum_{i=+,0,-} n_i^{\mathrm{id}}(T,\mu_i) = \frac{1}{2\pi^2}\sum_{i=+,0,-}\int_0^{+\infty} dk\, k^2 f_k(T,\mu_i) \quad (2)$$

And charge density:

$$n_{\mathrm{Q}}(T,\mu) = \left(\frac{\partial P}{\partial \mu}\right)_T = n_+^{\mathrm{id}}(T,\mu) - n_-^{\mathrm{id}}(T,\mu) \quad (3)$$

Where

$$f_k(T,\mu_i) = \left[exp\left(\frac{\sqrt{k^2+m^2}-\mu_i}{T}\right) - 1\right]^{-1} (4)$$

is a Bose-Einstein distribution function.

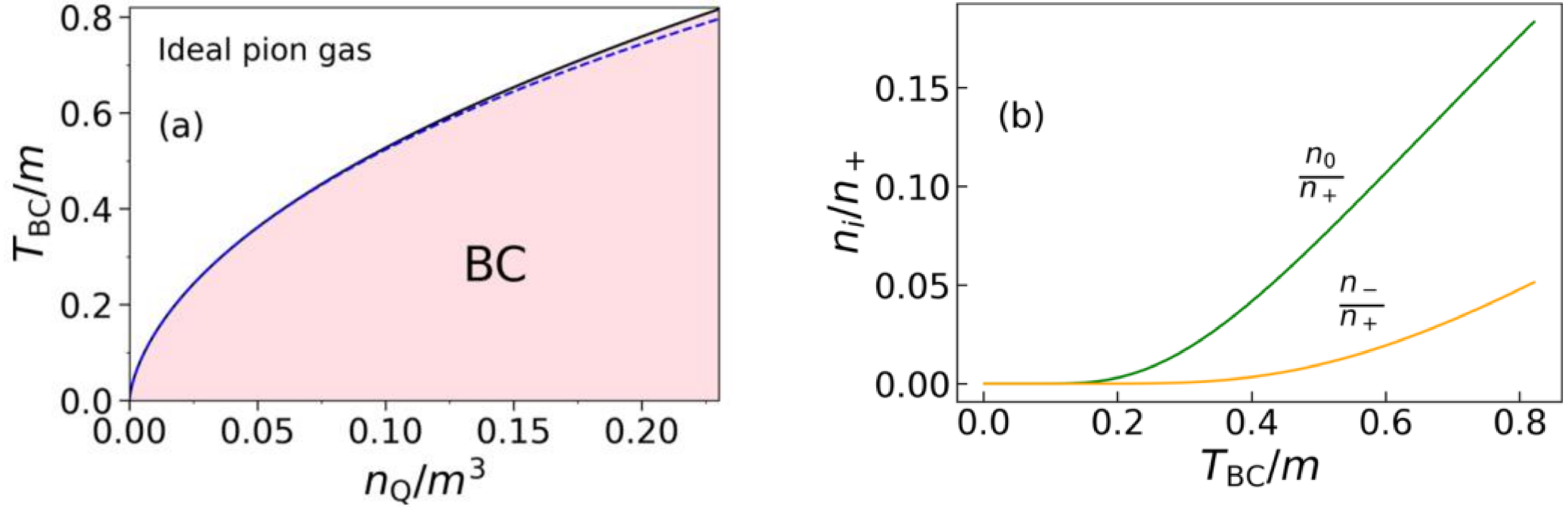
BOSE CONDENSATION

The inequality $|\mu| < m$ should be satisfied in the IdPG.

Under this line at $T < T_{\rm BC}(n_{\rm Q})$ there is a region with a nonzero BC of the π^+, and the total density now will be:

$$n(T,\mu) = \sum_{i=+,0,-} n_{\rm id}(T,\mu_i) + n_{\rm BC}^{+} \quad (5)$$

At $T = 0$, i.e., at zero temperature the pion system consists from the pure BC of π^+.



INTERACTING PION GAS

The self-consistency condition can be written as:

$$\left(\frac{\partial P}{\partial \mu^*}\right)_{m^*} = 0, \qquad \left(\frac{\partial P}{\partial m^*}\right)_{\mu^*} = 0 \ (6)$$

A. Mean-field model: for the system of interacting pions it's given by the following set of self-consistent equations:

$$P(T,\mu) = \sum_{i=+,0,-} p_{\mathrm{id}}(T,\mu_i^*) + \int_0^n dn' n' \frac{dU}{dn'} \ (7)$$

$$n(T,\mu) = \sum_{i=+,0,-} n_{\mathrm{id}}(T,\mu_i^*) + n_{\mathrm{BC}}^{+} \ (8)$$

$$\mu_i^* = \mu_i - U(n), \;\; U(n) = -An + Bn^2, \; A > 0, \; B > 0 \ (9)$$

Where $U(n)$ is the mean field that describes pion interactions.

B. Hybrid model: Another phenomenological model considered in our paper is constructed by combining the two frameworks:

$$P = \sum_{i=+,0,-} p_{\mathrm{id}}(T,\mu_i^*,m^*) + \frac{2}{3}Bn^3 - \frac{(m-m^*)^2}{2A} \ (10)$$

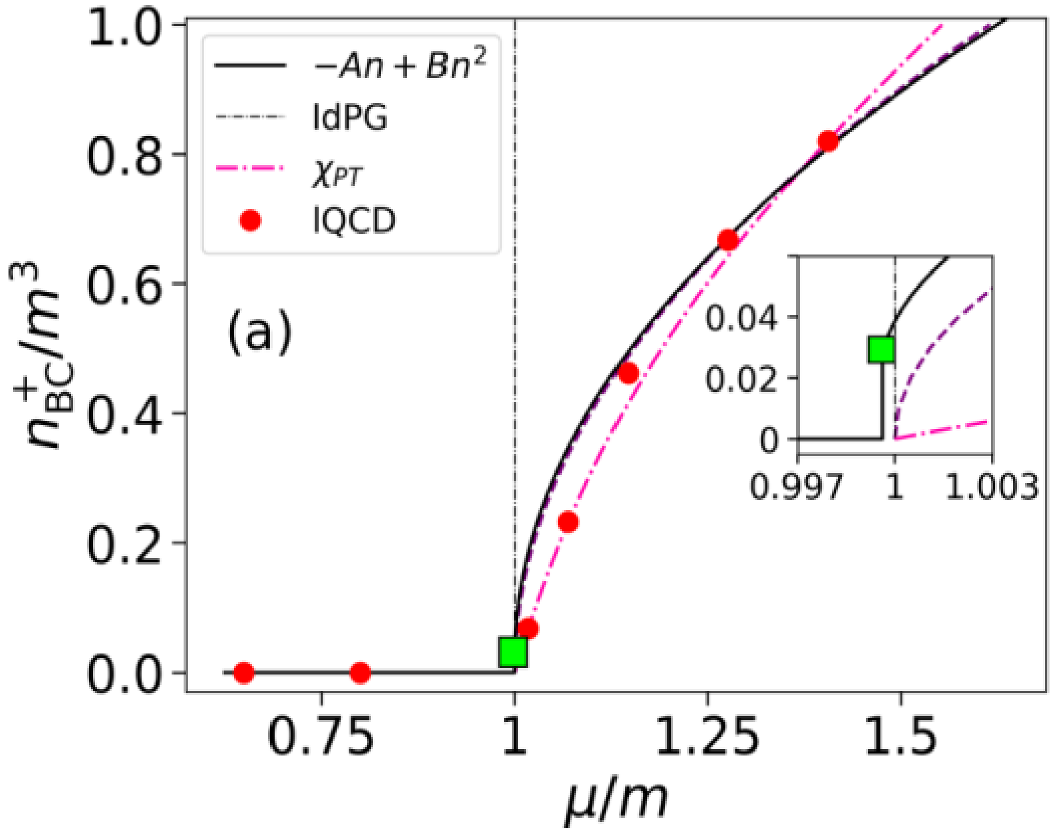
$$n(T,\mu) = \sum_{i=+,0,-} n_{\mathrm{id}}(T,\mu_i^*,m^*) + n_{\mathrm{BC}}^{+} \ (11)$$

$$\mu_i^* = \mu_i - Bn^2, \qquad m^* = m - An_s \ (12)$$

INTERACTING PION GAS

At $T = 0$, our models are same, so we can use same formulas here. A condition of the BEC, $\mu - U(n^+_{\rm BC}) = m$, leads to the following solution for $n^+_{\rm BC}$, that is useful for purposes of fitting:

$$n^+_{\rm BC}(T=0,\mu) = \frac{A+\sqrt{A^2+4B(\mu-m)}}{2B} \quad (13)$$



Here: $A = 0.05m^{-2}, B = 1.30m^{-5}$ (14)

Ground state density:

$$n_0 = \frac{3A}{4B} \approx 0.029m^3 \quad (15)$$

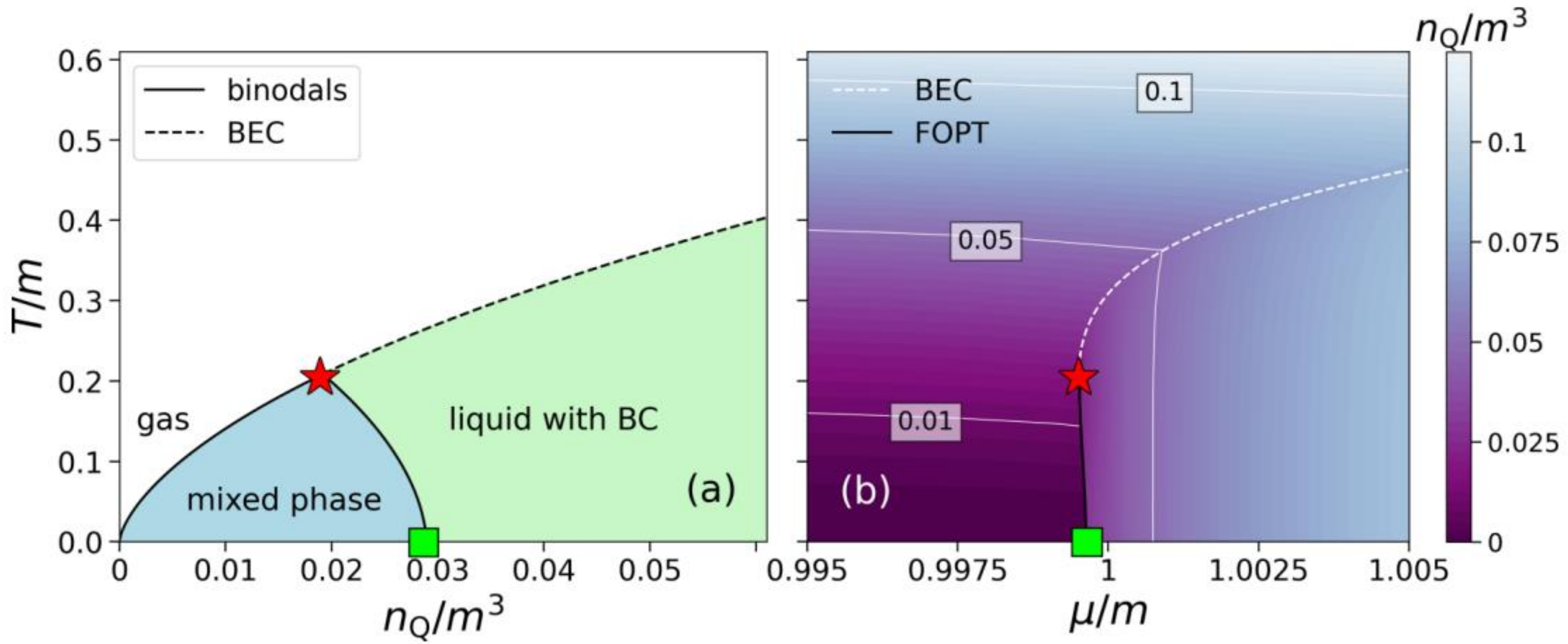
Chemical potential:

$$\mu_0 = m - \frac{3A^2}{16B} \approx 0.9996m \quad (16)$$

Ground-state binding energy per pion:

$$W = -\frac{3A^2}{16B} \approx -0.00036m \quad (17)$$

INTERACTING PION GAS AT $T > 0$



FLUCTUATIONS

From the known formula we can find the fluctuations of charge density in explicit way.

$$\omega_Q = \frac{T}{n_Q}\left(\frac{\partial n_Q}{\partial \mu}\right)_T \quad (18)$$

In the case of mean field model it give us simple analytical result for critical number density and chemical potential, where $\omega_Q \to +\infty$:

$$\omega_Q = \frac{T}{n_Q}\left[\frac{dU}{dn}\right]^{-1} + \frac{n_0\omega_0 + 4n_-\omega_-}{n_Q} \quad (19)$$

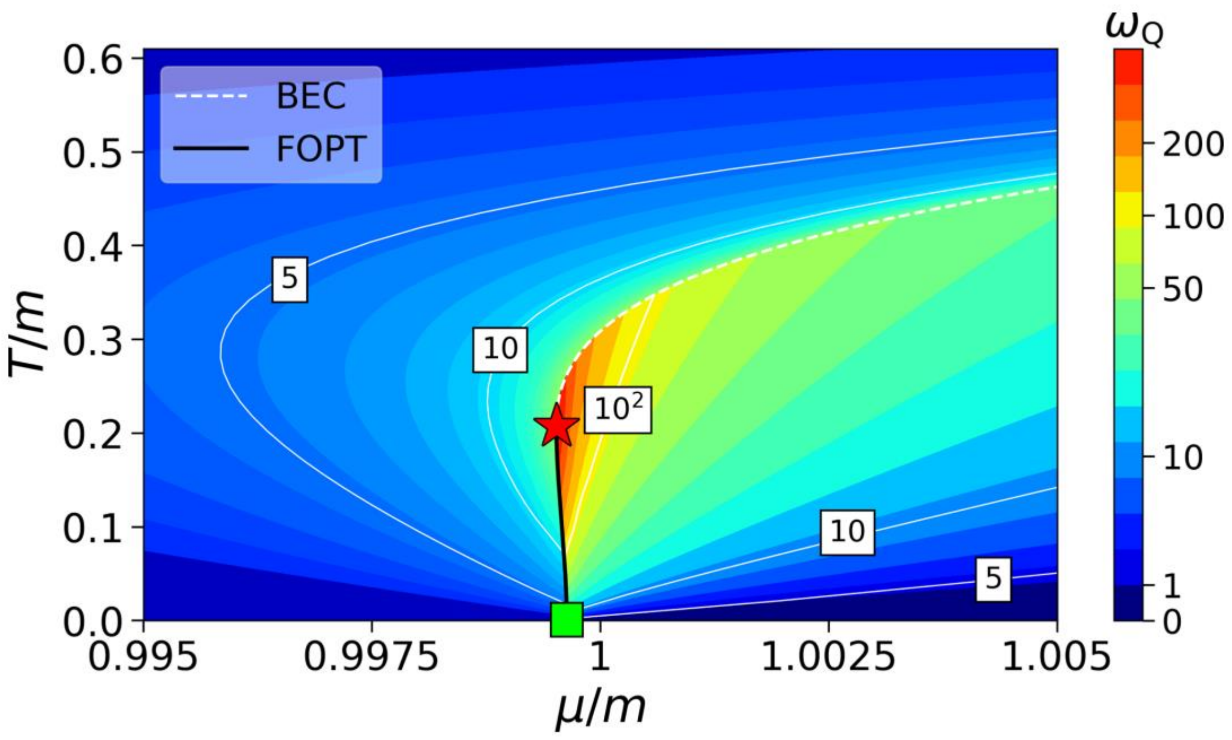
$$\frac{dU}{dn} = 0 \ \rightarrow n_c = \frac{A}{2B}, \ \mu_c = m - \frac{A^2}{4B} \quad (20)$$

In case of our parameters: $n_c \approx 0.019m^3, \ \mu_c \approx 0.9995m$

Numerically obtained critical temperature: $T_c \approx 0.208m$.

In the hybrid model:

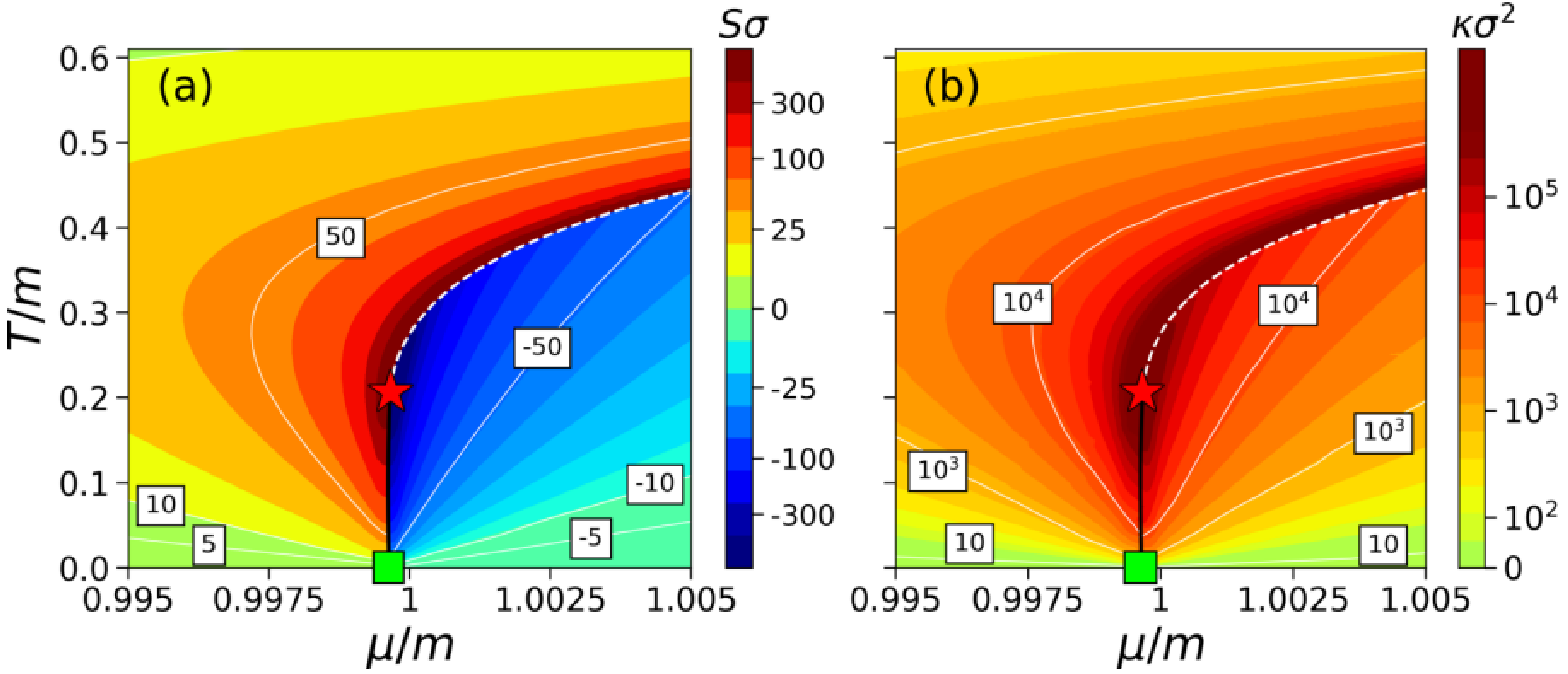
$n_c \approx 0.018m^3, \ \mu_c \approx 0.9996m, T_c \approx 0.206m.$



HIGHER ORDER FLUCTUATIONS

To study our hybrid model more precisely, let's introduce next well known quantities – skewness (left panel) and kurtosis (right panel)

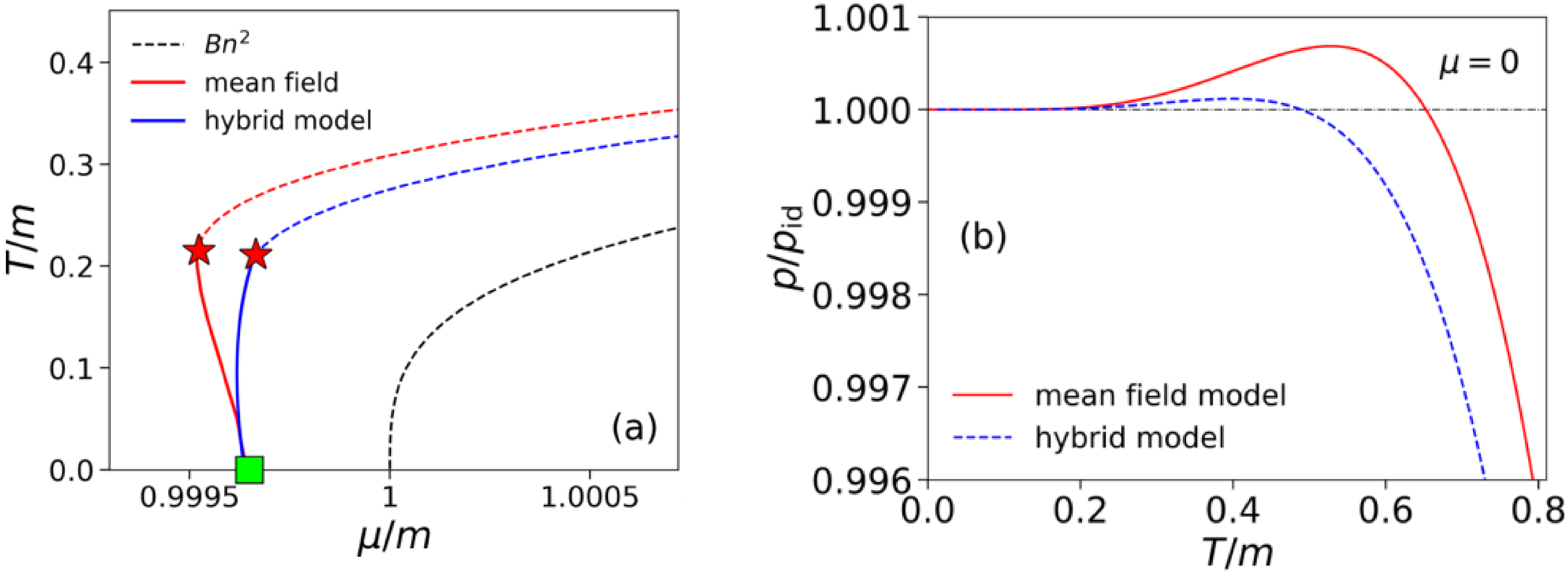
$$S\sigma = \frac{X_2}{X_1}, \quad k\sigma^2 = \frac{X_3}{X_2} \quad \text{where} \quad X_j = \frac{\partial^j (P/T^4)}{\partial(\mu/T)^j} \quad (21)$$



COMPARISON OF MODELS

One can see, that two considered models are close, so it is them results.

In order to find the difference skewness and kurtosis diagrams were build.

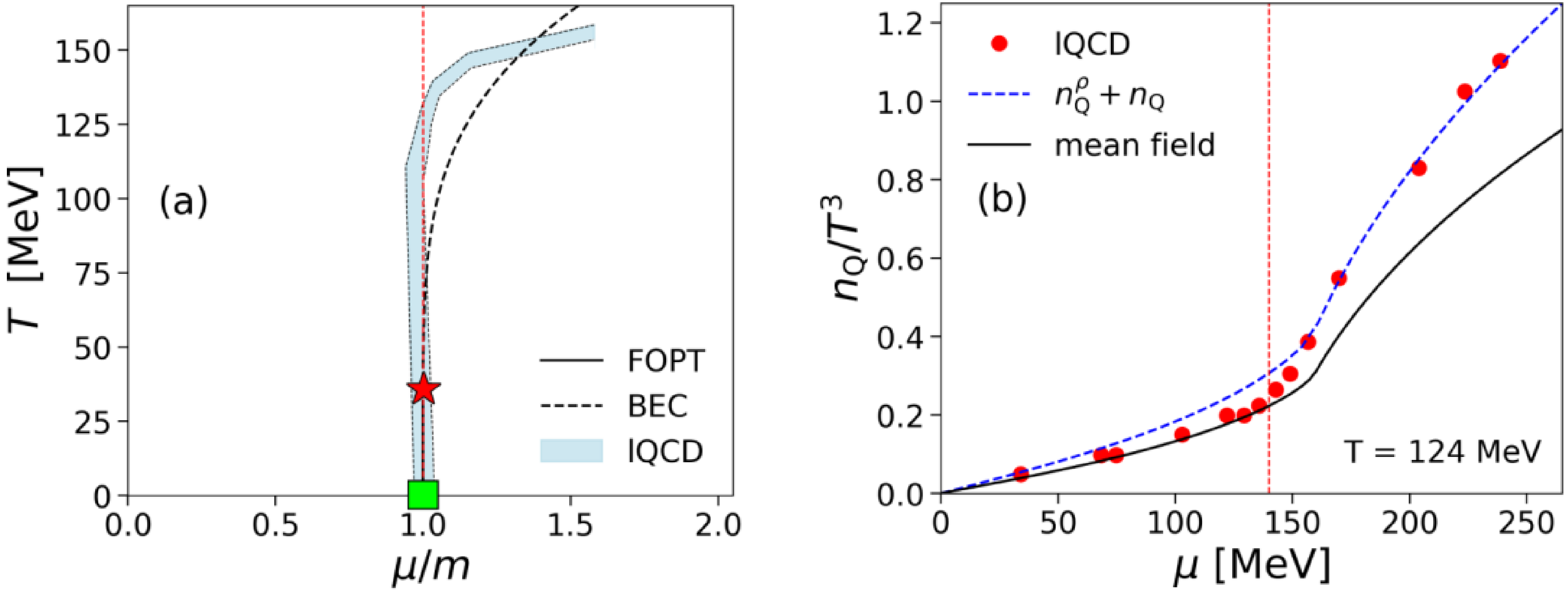


RESULTS AT $T \sim m$

Another interesting question: how our models behave at “high” temperature.

To estimate the resonance contribution to n_Q we calculate the n_Q^ρ value that comes from a presence of noninteracting $\rho_\pm$ mesons in the pion system:

$$n_Q^\rho = n_\rho^+ - n_\rho^- = \frac{3}{2\pi^2}\int_0^\infty k^2 dk\left[f_k(T,\mu,m_\rho) - f_k(T,-\mu,m_\rho)\right] \quad (22)$$



SUMMARY

- Thermodynamic properties of the interacting pion matter are studied in the two phenomenological models. Both models demonstrate the 1^{st} order phase transition with a position of the critical point at $\mu_c \cong m$ and $T_c \cong 28$ MeV. The line of Bose-Einstein condensation merges to the critical point.
- Unlike the Ideal Bose gas, fluctuations in considered models are finite during the Bose-Einstein condensation.
- The critical point in the system of interacting pions demonstrates a singular behavior of the electric charge fluctuations. These effects can be searched in the lattice QCD simulations.

THANK YOU FOR ATTENTION!

Questions?